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FRICTION CORRELATIONS FOR RECTANGULAR ROUGHNESSES

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Abstract—A relation for the roughness parameter R of the velocity profile in a fully rough flow is established as a function of the geometrical parameters of the roughness elements. The evaluation is based on a detailed analysis of experimental results described in the literature. Numerous data of friction factors and velocity profiles in channels with artificial rectangular wall roughnesses were taken into account. The geometrical parameters are the ratio of distance to height, the ratio of height to width of the roughness, and the ratio of the height of the roughness to the length of the velocity profile from the rough wall to the position of zero shear stress. Application of the relation determined in this way, which holds over a very broad range of geometrical parameters, allows the pressure drop to be calculated with a rather good accuracy in channels with rectangular wall roughnesses.

NOMENCLATURE

a_i,	coefficient and exponent, respectively;
D,	hydraulic diameter;
F,	flow cross section;
G,	geometry parameter;
h,	height of roughness elements;
h^+,	$= \frac{hu^*}{\nu}$, dimensionless roughness height, "roughness" Reynolds number;
L,	length of velocity profile between the wall and the zero-shear position;
p,	pitch of roughness elements;
R,	roughness parameter;
Re,	Reynolds number;
u^*,	friction velocity;
u^+,	dimensionless velocity;
w,	width of roughness elements;
y,	distance from the wall;
y^+,	$= \frac{yu^*}{\nu}$, dimensionless distance from the wall;
z,	coefficient of h/L-effect;
λ,	friction factor;
ν,	kinematic viscosity.

Subscripts

0,	related to $h/L = 0$;
k_1,	related to $h/w = k_1$;
k_2,	related to $p/h = k_2$.

1. INTRODUCTION

IT IS known that the heat transfer in a fully developed turbulent flow depends on the material properties (Prandtl number) of the fluid. Fluids with a low Prandtl number (such as liquid metals) have relatively good heat transfer properties. Fluids with a mean or high Pr-number (e.g. gases, water, organic liquids) frequently require technical measures to improve the heat transfer so that the heat transferring equipment can be kept small. One method frequently used for this purpose, which has been suggested also for the fuel elements of the gas cooled fast breeder [1], consists in artificially roughening the heat transferring wall of the flow channel. In that case the efficiency with respect to the heat-transfer properties of the roughness is a function of the shape and arrangement of the roughness elements.

Since the roughness of the wall, as a consequence of the higher turbulence produced in the flow, not only increases the heat transfer but also produces additional losses of pressure, it is meaningful to use optimum geometrical shapes and arrangements of the artificial roughness, i.e. to generate no more turbulence than is necessary to reduce the wall temperature.

However, this requires detailed knowledge of the relation between the efficiency and the geometry of the wall roughness.

An important parameter, which also determines the loss of pressure, is the roughness parameter R which can be determined from isothermal pressure drop measurements or from measurements of the velocity distribution. Below, this roughness parameter R will be investigated in more detail for rectangular roughnesses.

Under the assumption of universal velocity profiles at rough channel walls according to Nikuradse [2]

$$u^+ = 2{\cdot}5 \ln \frac{y}{h} + R \tag{1}$$

the friction factor λ can be represented as a function of the geometry parameter G, the relative thickness of the flow layer L/h, and the roughness parameter R [3]:

$$\sqrt{\left(\frac{8}{\lambda}\right)} = 2{\cdot}5 \ln L/h + R - G. \tag{2}$$

The roughness parameter R is a function of the roughness geometry.

A large number of results from measurements of the pressure drop can be seen from the literature. Despite

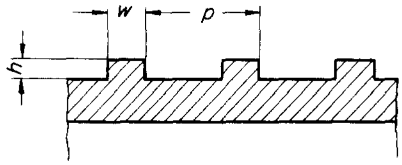

FIG. 1. Geometrical parameters of rectangular roughness geometry.

the wealth of experimental data, there is as yet no general relation between the roughness parameter R and the quantities describing the geometry of rectangular roughnesses. One important reason for this absence may well be the necessity to describe the geometry of rectangular roughnesses by three parameters (Fig. 1): the height of the roughness h, the width of the roughness w, and the pitch of roughness elements p. This renders the determination of any influence of the individual parameters from the measured data very difficult, because there are no systematic studies. Another difficulty lies in the fact that the studies have been conducted on such different channels as circular tubes, parallel plates, rectangular channels, open troughs and, very frequently, on annular gaps with rough inside tubes.

Accordingly, it is not surprising to see that Maubach [4] for the first time attempted to arrange the wealth of data by some system. The method he developed [3] allowed a separation to be made of the influence of channel geometry and roughness geometry by means of the roughness parameter R. Although the extrapolation from one channel to another of experimental results, e.g. of friction factors of the flow layer influenced by roughnesses, had been possible before by means of the Hall transformation [5], studies of Wilkie *et al.* [6] on a plate channel with identical and non-identical roughnesses indicated that the assumption of a coincidence of maximum velocity and zero shear stress for the application of the Hall transformation had been wrong. This wrong assumption had the result that the transformed friction factors obtained from annular gaps with rough core tubes had to be assigned correction factors when extrapolated to rod bundles [7]. Besides, such integral parameters as the friction factor and the hydraulic diameter do not seem to lend themselves well to the extrapolation of measured results. A quantity such as the roughness parameter R, which also exists in the velocity profile, should be more useful for this purpose.

On the basis of the results he used, Maubach [4] was able to indicate a dependence of the roughness parameter R on the dimensionless geometry parameters p/h and h/w in the form of a diagram. Meerwald [8] divided the range of p/h and h/w values investigated, indicating diagrams for different ranges of p/h and h/w from which the dependence of the roughness parameter on p/h and h/w, respectively, can be seen.

The analysis of Wilkie's pressure drop results [6], which had been measured on a rectangular channel with identical and non-identical wall roughnesses, indicated that the height of the roughness h relative to the length L of the velocity profile from the rough wall to the position of zero shear stress has a major influence upon the magnitude of the roughness parameter $R(h^+)$ [9].

No dependence of the roughness parameter upon the relative roughness height h/L had been apparent from Nikuradse's studies [2] on circular tubes (L = tube radius) with sand roughnesses, aside from the results for a relative roughness $L/h = 507$. For this roughness, Nikuradse found a deviation and suspected that "most probably the condition of geometric similarity had not been met". In the range of fully rough flow where the pressure drop increases in proportion to the square of the mean velocity, Nikuradse was able to describe the measured velocity profiles quite well by equation (1) or

$$u^+ = 2{\cdot}5 \ln y^+ + R - 2{\cdot}5 \ln h^+ \qquad (3)$$

which indicated the dependence on the roughness height. However, a detailed analysis of Nikuradse's values indicates that the last term on the right hand side of equation (3) allows only an approximate description to be given of the dependence of the height of roughness, because the roughness parameter R is not completely independent of h/L. Since in Nikuradse's studies there had not been a complete geometrical similarity, a dependence of the roughness parameters on h/L does not come as a surprise. Unfortunately, however, in most of the following studies of artificial roughnesses it was assumed that the dependence on the height of the roughness would be the same as in the case of Nikuradse's sand roughness also in all the other kinds of roughnesses. A literature survey of artificial roughnesses [10] as well as more recent measurements of rectangular roughnesses carried out by Dalle Donne and Meerwald [11] show that in general R is a function of h/L. That the roughness parameter R, in the case of full geometric similarity, is independent of the scale of the test sections chosen is proved by the investigations by Möbius [12] who, however, also varied the pipe diameter on the same scale as the roughness and in this way met the condition of geometric similarity. In this way, another geometrical parameter, namely the length of the velocity profile over the roughness up to the point of zero shear stress, must be taken into account in evaluating the data in the literature. Only Dalle Donne and Meerwald [11] have so far taken into account all the parameters and given relations for the roughness parameter R over a large area of geometry parameters.

The determination of relations generally valid for the roughness parameter of rectangular roughnesses further becomes more difficult by the fact that roundings or chamfers of roughnesses, which may also be caused by fabrication tolerances, bring about a considerable scatter of the roughness parameters (± 20 per cent), as has been determined by Meerwald [8] on the basis of studies conducted by White and White [13].

Finally, it must be said that the quality of the measured data varies. Some results are contradictory, others are difficult to interpret because the experimental conditions are not always indicated too clearly. Despite these difficulties and shortcomings in the data material at hand an evaluation of all the data measured seems to be necessary so that the relations of the pressure drop behaviour caused by artificial rectangular roughnesses can be determined with greater accuracy and in this way further experimental studies can either be avoided or carried out more specifically, as has been requested also by Sabersky [14].

2. METHOD OF EVALUATION

2.1. *Basic conditions*

In order to allow a comparison to be made between the results obtained from different experimental geometries, all the data must be based on the same assumptions. The pressure drop results published in the literature are indicated either for "root" or "tip" or other diameters of the roughness elements. Since different reference dimensions of flow channels give rise to different values of the friction factor, this study consistently uses the volumetric diameter which is produced when the roughnesses are molten. Accordingly, the length of the velocity profile is obtained as the distance between the zero shear stress line and the volumetrically averaged wall level. These definitions are based on the fundamental work by Nikuradse [1] and Schlichting [15]. Accordingly, all the friction factors and Reynolds numbers have been changed to volumetric dimensions if necessary.

The roughness parameters are determined from all the measured results on the same hypothesis, that is, the assumption of universal velocity profiles. For studies on rough circular tubes the roughness parameter R according to equation (1) can be determined from velocity profile measurements or, according to equation (2), from pressure drop measurements. For annular gaps and rectangular channels the zero shear stress method developed by Maubach is used which is described in more detail in [4]. It is based on the assumption of two velocity profiles starting from opposite walls, equation (1) applying to a rough wall, and

$$u^+ = 2{\cdot}5 \ln y^+ + 5{\cdot}5 \tag{4}$$

applying to a smooth wall. The intersection of both profiles is regarded as the zero shear stress position. Maubach's method is applied because of its simplicity. In addition, the results obtained in this way are in good agreement with the values calculated by means of the empirical method by Warburton and Pirie [16]. More recent studies on asymmetrical flow distributions and smooth walls in addition show that the deviations from the law of the wall (equation 4) are not as great as has frequently been assumed [17]. Since, in addition, a law of the wall has been measured in a good approximation in previous studies of rough walls [18–20], Maubach's method is justified also as far as the underlying assumptions are concerned.

As a further constraint only those data were used for evaluation for which there was a "roughness" Reynolds number $h^+ = hu^*/\nu \geqslant 100$. This is regarded as the lower limit of fully rough flow where the resistance changes in proportion to the square of the velocity and thus $R =$ const. applies, although the roughness parameters R were not constant in a few experimental results even above these limits [20].

All the results known to the authors were used for evaluation. Some experimental results have been published only as transformed friction factors and transformed Reynolds numbers. Of course, such data have been incorporated only if the method of transformation was known in all its details and the necessary data for re-transformation were indicated. Unfortunately, this is not the case with most transformed values so that, as a consequence, valuable information is lost.

2.2. *Procedure*

The study served the purpose of finding relations which would allow a calculation of the roughness parameter R to be made from the given roughness geometry and the thickness of the flow layer.

In dimensionless quantities this relation can be set up as follows:

$$R = f(p/h, h/w, h/L). \tag{5}$$

The dependence of the roughness parameter on the relative height of roughness h/L is taken into account by a polynomial setup which establishes a relation between R at the respective h/L and a fictitious R_0 for $h/L \to 0$. For certain reference values $h/w = k_1$, $p/h = k_2$, let the following relation apply:

$$R_{k_1,k_2} - R_{0_{k_1,k_2}} = \sum_{k=2}^{n} z_k (h/L)^{k-1}. \tag{6}$$

For random values of h/w and p/h it then follows:

$$R - R_0 = \frac{R_0}{R_{0_{k_1,k_2}}} \sum_{k=2}^{n} z_k (h/L)^{k-1}. \tag{7}$$

The polynomial setup deviates from the form proposed in [10], but this turned out to be more useful for the evaluations described in this study. Introduction of the parameter R_0 reduces the problem to the following form:

$$R_0 = f(p/h, h/w). \tag{8}$$

All calculations were carried out on an IBM 370/165 computer. For the rest of the evaluation the h/L dependence of the R-values was neglected initially. By means of a least-squares fit (LSF) the data were approximated over h/w and p/h by second- and third-order polynomials, respectively, and arranged in a plane value matrix W. The grid of the matrix was locally adapted to the distribution density of the measured points. The matrix used for evaluation in this study had the dimensions of $W(p/h; h/w) = (39; 6)$. Along with the value matrix W a weight matrix G was generated. In this matrix each element G_{ik} indicates the weight of the assigned element W_{ik} of the value matrix. The weight in this case corresponds to the number of measured points covering the respective element of the value matrix.

A combined evaluation of the value and weight matrices by means of the LSF-approximation generated second-order polynomials in h/w and third-order polynomials in p/h. It was attempted by means of

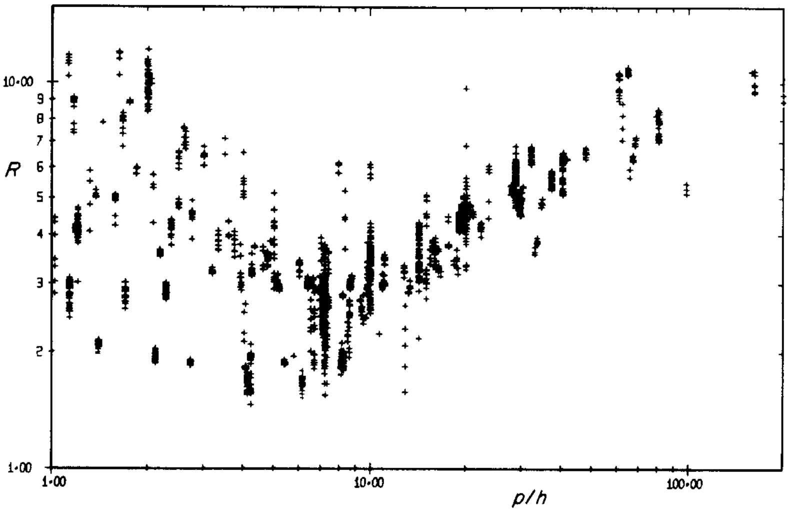

FIG. 2. Untransformed roughness parameter R as a function of p/h.

higher order polynomials to achieve a better adaptation to the measured data. The results were unsatisfactory, however, because no consistent dependencies were found. By means of the polynomials determined it is possible to transform all the measured points to an arbitrary normalization geometry $h/w = k_1$, $p/h = k_2$. The measured points referred to the normalization geometry in a first approximation furnish the dependence of the roughness parameter R upon the relative thickness of the flow layer h/L which was accounted for by a second-order polynomial by means of an LSF-approximation. The dependence is a first-order dependence because initially a value matrix was used for transformation of the measured points which was generated from R-values not adjusted with respect to h/L. The h/L dependence was then improved by iteration until the adjusted R-values, converted to the normalization geometry, according to the value matrix generated, showed h/L dependence below a given threshold.

3. RESULTS AND DISCUSSION

The evaluation covered the data by a total of 35 authors [6, 8, 11, 12, 15, 19–48] with approximately 2500 measured points for rectangular roughnesses of $h^+ \geqslant 100$. A survey of all the test sections and the values used for the roughness parameter can be seen from [49]. The data base available covers the following ranges of geometry parameters:

$$0{\cdot}35 \leqslant p/h \leqslant 196$$
$$0{\cdot}02 \leqslant h/w \leqslant 15{\cdot}1$$
$$0{\cdot}008 \leqslant h/L \leqslant 0{\cdot}997.$$

Figure 2 shows all the values used, referred to volumetric channel dimensions, as a function of the dimensionless distance p/h. Especially for small dimensionless rib distances p/h the measured values scatter very broadly.

3.1. *Results of evaluation*

If the method outlined above is applied to all the measured values, it is possible to represent by a family of curves the geometry dependence of the roughness parameter R_0 of p/h and h/w (Fig. 3), which family of curves is determined by the matrix generated. The roughness parameters are represented over the dimensionless rib distance with the height-to-width ratio h/w as a parameter. The values of the roughness parameter apply to the fictitious value $h/L \rightarrow 0$, i.e. to an infinitely thick flow layer above the rough surface. Within the range of $0{\cdot}3 \leqslant h/w \leqslant 8$ represented the roughness parameter turns out to be the smaller, the larger h/w or, to put it differently, the friction coefficient for the flow over a rough surface is the higher, the thinner the roughness element is. This dependence on the geometry of the individual element becomes weaker at large distance ratios p/h. The smallest value of the roughness parameter in each case shifts towards a smaller p/h as h/w increases. In the range of $0{\cdot}3 \leqslant h/w \leqslant 15$ the minimum of the roughness parameter can be described by the following equation:

$$(p/h)_{R=\min} = 9{\cdot}9(h/w)^{-0{\cdot}345}. \tag{9}$$

If all the measured values used are standardized to a fixed value of $h/w = k_1 = 1{\cdot}4622$ by means of the matrix generated, Fig. 4 will result. A comparison of Fig. 4 with Fig. 2 clearly shows that the scattering range of the measured points is greatly reduced. On the other hand, it must also be seen that there is still a considerable amount of scatter, which is due to the uncertainties mentioned above. The value of $h/w = 1{\cdot}4622$ resulted as a mean value in the h/w range of measured values. All the measured points represented were already reduced by the dependence on h/L determined during evaluation.

This dependence on the ratio between the height of roughness and the length of the velocity profile is represented in Fig. 5. For this purpose all the measured points were normalized not only to $h/w = 1{\cdot}4622$, but also to $p/h = k_2 = 10$. The second-order polynomial obtained by LSF-approximation has a maximum at $h/L = 0{\cdot}38$. It is described by the following equation:

$$R_{k_1,k_2} = 2{\cdot}900 + 1{\cdot}490h/L - 1{\cdot}972(h/L)^2. \tag{10}$$

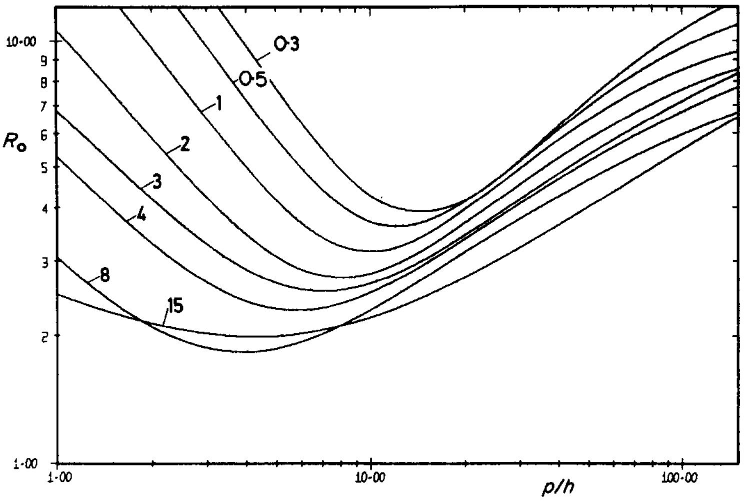

FIG. 3. Roughness parameter R_0 transformed to $h/L = 0$ as a function of p/h for different h/w.

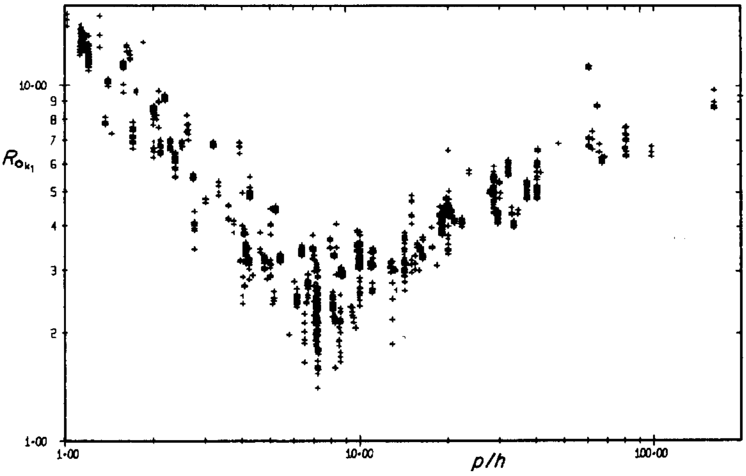

FIG. 4. Roughness parameter $R_{0_{k_1}}$ transformed to $h/L = 0$ and $h/w = k_1 = 1{\cdot}4622$ as a function of p/h.

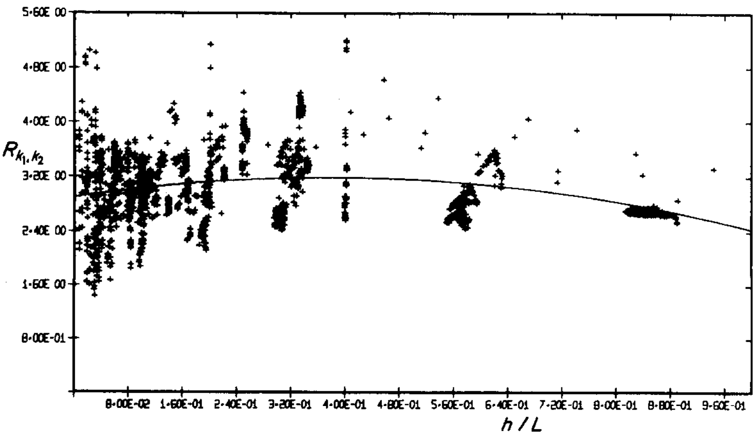

FIG. 5. Roughness parameter R_{k_1,k_2} transformed to $h/w = k_1 = 1{\cdot}4622$ and to $p/h = k_2 = 10$ as a function of h/L.

As is evident from the figure, the dependence of the roughness parameters on the ratio between the height of roughness and the length of profile is relatively weak in rectangular ribs as against other types of roughness [10]. However, it should be borne in mind that the absolute value of this dependence changes with the magnitude of the roughness parameter because all the values are normalized. That the curve of equation (10) as shown in Fig. 5 is not a random result, as could be expected in the light of the marked scatter of the measured points, is evident from the detailed analysis of the results of individual authors. In an analysis of this type random errors are largely excluded and it is seen that low h/L values are accompanied by an increase, high values by a decrease of R [28, 35, 44], which is also seen from Nikuradse's studies on sand roughnesses [2].

The curve passing through a maximum can probably be explained by two opposed effects both of which are connected with the definition of the volumetric diameter agreed upon. This agreement fixes the origin of the velocity profile. The velocity profile over a rough wall is assumed to be a mean value of the velocity distribution right over the rib and in the spaces in between. Now, if the height of the rib increases relative to the length of the velocity profile, it is easy to see that the true origin of the averaged velocity profile is shifted in the direction of the flow. This means that the dimensionless velocity profiles according to equation (1) are raised because of the wall distance y, which will then be smaller, and the roughness parameter R will accordingly be higher, which is actually found to be the case. An opposite effect is encountered at very high h/L-values. Between the ribs there will then be a dead water area. These flow influences are not taken into account by the definition of a volumetric diameter, and the friction factor as determined from measurements will be too high because of the flow cross-section and the hydraulic diameter having been assumed too high as a consequence of

$$\lambda \sim F^2 D. \tag{11}$$

This excessive friction factor in turn gives rise to a roughness parameter R according to equation (2) which is too small. There would probably be no dependence of the roughness parameter R on h/L if the diameter and the flow cross-section and thus also the origin of the velocity profile could always be adapted to the existing flow conditions, which, however, is not possible for obvious reasons.

3.2. *Application of results*

In order to be able to apply the dependences determined to practical problems in a simple way, the family of curves represented in Fig. 3 is indicated as an equation. The approximation then found is

$$R_0 = a_1(p/h)^{a_2} + a_3(p/h)^{a_4} \tag{12}$$

with

$$a_1 = 18{\cdot}5(h/w)^{-0{\cdot}947}$$
$$a_2 = -1{\cdot}143(h/w)^{-0{\cdot}147}$$
$$a_3 = 0{\cdot}33(h/w)^{0{\cdot}1483}$$
$$a_4 = 0{\cdot}758(h/w)^{-0{\cdot}11}$$

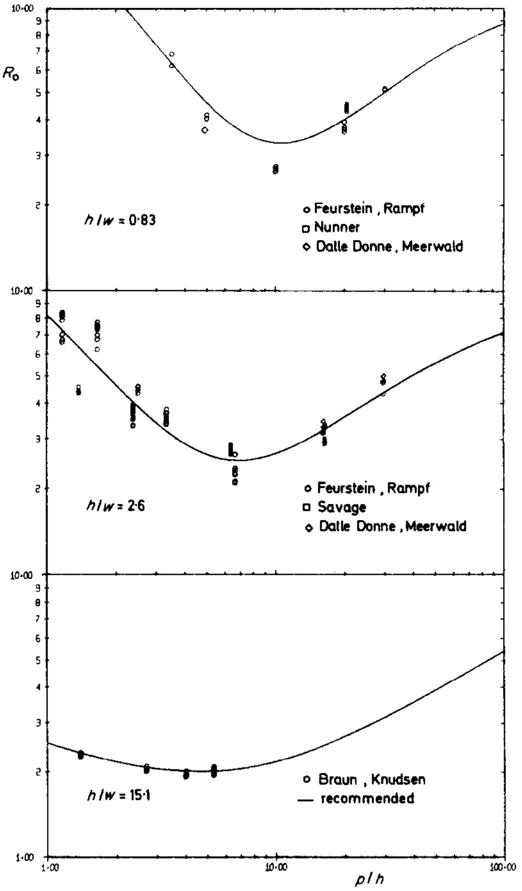

FIG. 6. Comparison between calculated and measured results: roughness parameter R_0 as a function of p/h for 3 values of h/w.

with the following range of validity:

$$1 \leqslant p/h \leqslant 40$$
$$0{\cdot}3 \leqslant h/w \leqslant 8$$
$$R_0 \leqslant 10.$$

The maximum deviation of the values calculated according to equation (12) from the curves determined from the matrix is 8 per cent.

If the friction coefficient is wanted for a given roughness, it is possible to calculate the roughness parameter R_0 for $h/L = 0$ according to equation (12). For a given h/L value one then obtains the roughness parameter as:

$$R = R_0 + \frac{R_0}{R_{0_{k_1,k_2}}}(R_{k_1,k_2} - R_{0_{k_1,k_2}}). \tag{13}$$

In order to facilitate the application, this method will now be explained on the basis of a simple example. Let the friction coefficient be wanted of a rough tube of the dimensions $D_{vol} = 100\,\text{mm}$; $h = 10\,\text{mm}$, $w = 20\,\text{mm}$ and $p = 40\,\text{mm}$ for a fully rough flow. According to equation (12), one finds for $h/w = 0{\cdot}5$ and $p/h = 4$ the value of $R_0 = 7{\cdot}10$ and from equation (10) for $h/L = 0{\cdot}2$ the value of $R_{k_1,k_2} = 3{\cdot}12$. Accordingly, from equation (13), the roughness parameter turns out to be $R = 7{\cdot}10 + 7{\cdot}10/2{\cdot}90(3{\cdot}12 - 2{\cdot}90) = 7{\cdot}64$. From equation (2) with $G = 3{\cdot}75$ for rough tubes [3] one then finds $\sqrt{(8/\lambda)} = 2{\cdot}5 \ln 5 + 7{\cdot}64 - 3{\cdot}75$ or $\lambda = 0{\cdot}1277$.

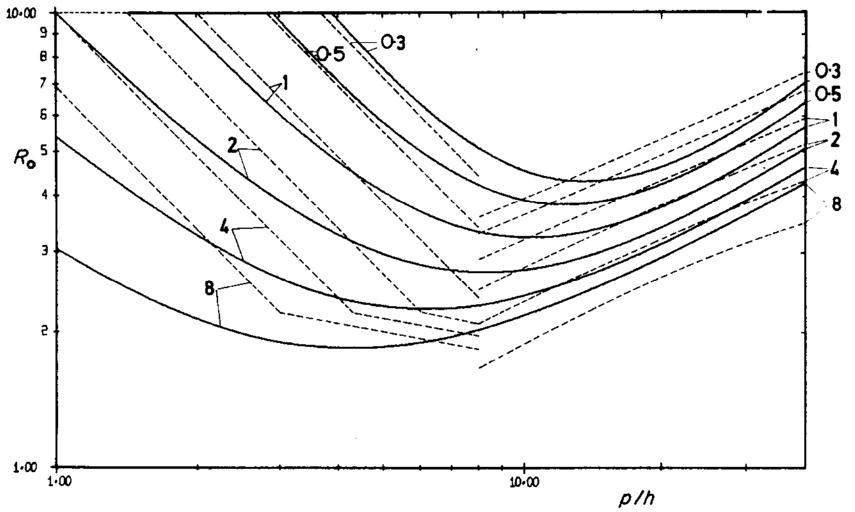

FIG. 7. Roughness parameter R_0 as a function of p/h for different h/w. —— according to equation (12); ---- recommended by Dalle Donne and Meerwald [11].

3.3. *Comparison with measured values and relations from the literature*

Figure 6 shows a comparison of the measured values from the literature, which were adjusted for the h/L effect according to equation (10), with the dependence on p/h of the roughness parameter as developed from all the measured values for three values of h/w. All the values measured within $(0{\cdot}96-1{\cdot}04)h/w$ were taken into account. Agreement between the measured values and the shapes of the curves is good if one takes into account the numerous possibilities of errors. When considering the amount of scatter of the measured points one must also bear in mind that a specific measuring error of the friction factor will result in a major error in the roughness parameter. This in particular applies to low values of R, i.e. to marked roughnesses, as can easily be verified by applying equation (2).

Since the literature contains only very few general statements about the dependence of the roughness parameter on the geometry of the individual roughness, only the data by Dalle Donne and Meerwald [11] will be used for comparison with the relations found in this study (Fig. 7). In each case the roughness parameters adjusted for h/L are compared, equation (12) being used for the new relations. For small p/h and small h/w the data by Dalle Donne and Meerwald are in rather good agreement with the relations found, but there are major deviations for large h/w with small p/h. The reason may be that the new relations are based on a larger number of measured values, especially in this range. However, rectangular ribs in this range are not the right method of increasing the heat transfer, because a large dead water area will be generated between the ribs which greatly impedes the heat transfer. What is more important for an improvement of heat transfer by artificial roughnesses is the area of the minimum values of the roughness parameter ($8 < p/h < 12$). However, in this area the data by Dalle Donne and Meerwald also deviate quite considerably. Above all, the inconsistency at $p/h = 8$ is disturbing. At high p/h values the new data cover a smaller range of R. This seems to be reasonable because the influence of the individual rib will certainly be reduced at large distances. Obviously, the reason for the large amount of scatter in the data according to [11] must be the fact that the h/L dependence assumed in [11] is too high so that R-values are generated in data reduction, especially at high h/w, which are too small. If the new relations are plotted versus the roughness geometry parameter p^2/hw introduced by Dalle Donne and Meerwald [50], the large amount of scatter of the curves indicates that this parameter p^2/hw is unsuitable for correlating measured values of rectangular roughnesses [49]. On the whole, Fig. 7 demonstrates the progress that has been achieved as a result of this study in describing the geometry dependence of the roughness parameter.

4. CONCLUSIONS

This analysis of all the results obtained from measurements on rectangular roughnesses known to the authors has shown that it is possible to indicate a general relationship of the roughness parameter R as a function of geometry for a fully rough flow. The relationship found applies over a very broad range of the three geometry parameters; these parameters are the ratio between height and width h/w, the ratio between distance and height p/h, and the ratio between the height of the roughness and the length of the velocity profile h/L. Evaluation of the measured results requires agreement to be reached on the channel dimensions. In this study the volumetric diameter was used throughout.

In order to allow a comparison to be made between the results measured in different flow channels, the same method of transformation must consistently be used. In this case, this is Maubach's method.

The pressure drop in channels with rectangular roughnesses can be calculated on the basis of the relations found. The accuracy of these relations is a function of the geometrical tolerances of the roughness elements, the quality of the data measured in the individual studies, the method of transformation and, finally, the assumption that the friction factor is independent of Reynolds number applies to a sufficiently high "roughness Reynolds number" h^+ (in this case $h^+ \geqslant 100$). For better accuracy of the relations indicated further systematic experimental studies will certainly have to be carried out over a broad range of h^+ and in the transition range between a hydraulically smooth and a fully rough flow. Such studies could be very valuable for the application of artificial roughnesses in heat transfer equipment.

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CORRELATIONS POUR LE FROTTEMENT SUR RUGOSITES RECTANGULAIRES

Résumé—On établit une relation pour le paramètre de rugosité R du profil de vitesse dans un écoulement en régime rugueux en fonction des paramètres géométriques des éléments rugueux. L'évaluation est basée sur une analyse détaillée des résultats expérimentaux décrits dans les publications. De nombreuses données relatives aux coefficients de frottement et aux profils de vitesse dans des conduites à rugosités artificielles rectangulaires ont été prises en compte. Les paramètres géométriques sont le rapport de la distance entre rugosités à leur hauteur, le rapport hauteur sur largeur des rugosités et le rapport de la hauteur d'une rugosité à la distance de la paroi rugueuse au point d'annulation de la contrainte de cisaillement turbulent. L'application de la relation obtenue par ce moyen, qui est valable dans un domaine très étendu de paramètres géométriques, permet de calculer avec une précision assez bonne la perte de charge dans des conduites avec rugosités rectangulaires.

REIBUNGSBEZIEHUNGEN FÜR RECHTECKIGE RAUHIGKEIT

Zusammenfassung—Es wird eine Beziehung für den Rauhigkeitsparameter R des Geschwindigkeitsprofils in einer vollständig rauhen Strömung als Funktion der geometrischen Parameter der Rauhigkeitselemente aufgestellt. Die Gleichung basiert auf einer detaillierten Auswertung experimenteller Ergebnisse aus der Literatur. Zahlreiche Reibungszahlen und Geschwindigkeitsprofile in Kanälen mit künstlich aufgebrachten, rechteckigen Wandrauhigkeitselementen werden in die Rechnung mit einbezogen. Die geometrischen Parameter sind das Verhältnis des Abstands zur Höhe und das Verhältnis der Höhe zur Breite der Rauhigkeitselemente, weiterhin das Verhältnis der Höhe der Rauhigkeitselemente zum Abstand zwischen der rauhen Wand und der Stelle mit der Schubspannung Null. Die Anwendung der auf diese Weise aufgestellten Beziehung, die einen großen Bereich geometrischer Parameter umfaßt, macht es möglich, den Druckabfall in Kanälen mit rechteckigen Wandrauhigkeiten zu berechnen.

КОРРЕЛЯЦИЯ ТРЕНИЯ ДЛЯ ПРЯМОУГОЛЬНЫХ ЭЛЕМЕНТОВ ШЕРОХОВАТОСТЕЙ

Аннотация — Установлена зависимость параметра шероховатости и профиля скорости в полностью возмущенном потоке от геометрических параметров элементов шероховатости. Вычисления основываются на подробном анализе экспериментальных результатов, описанных в литературе. Учитываются многочисленные данные по коэффициентам трения и профилям скорости в каналах с искусственно нанесенными прямоугольными шероховатостями на стенке. Геометрическими параметрами являются отношение расстояния между шероховатостями к ее высоте, отношение высоты к ширине шероховатости и отношение высоты шероховатости к толщине пограничного слоя, отсчитываемой от шероховатой стенки до положения нулевого касательного напряжения.

Применение полученной зависимости, которое справедливо для очень широкого диапазона геометрических параметров, позволяет рассчитать с достаточно хорошей точностью падение давления в каналах с прямоугольными шероховатостями на стенке.